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IDENTIFYING CHANGE IN SECONDARY MATHEMATICS TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE

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Like several other research groups, we have been investigating measures for capturing change in middle and high school teachers' mathematical pedagogical content knowledge (PCK). This report focuses on 14 teachers who have completed a distance-delivered master's degree in mathematics education. The group is the first of five cohorts who will complete a program that seeks to develop content proficiency, intercultural competence, and pedagogical expertise for teaching mathematics. Analysis included pre- and post-program data from observations of participants at work and written PCK assessments. Results indicate significant changes in curricular content knowledge and discourse knowledge. Path analyses suggest teacher discourse knowledge as measured by the written assessments is significantly related to discourse knowledge as measured by the post-program observation.

Key words: Pedagogical content knowledge, in-service teachers, professional development

Background

In response to the call for advanced professional education accessible to in-service teachers, the Mathematics Teacher Leadership Center (Math TLC), an NSF-funded Mathematics and Science Partnership project has developed and is researching a virtual master's program in mathematics education. The primary goals of the program are to develop content proficiency, cultural competence, and pedagogical expertise for teaching secondary grades mathematics (grades 6 to 12). To document the development of mathematics teaching expertise, project research investigates the pedagogical content knowledge of teacher participants before their enrollment into and after their completion of the master's degree program. Earlier reports have offered preliminary results on data collected mid-program. This report is the first to include pre- and post-program data for the first cohort of graduates.

Pedagogical content knowledge (PCK) is a construct described by Schulman (1986) and subsequently refined by other researchers to encompass the unique collection of discipline-connected knowledge needed for teaching. As PCK has become widely utilized in research on teacher development, the idea of "mathematical knowledge for teaching" has emerged as a useful construct (Ball, Hill, & Bass, 2005). In particular, in seeking to capture what elementary grades teachers do in the teaching of mathematics, the focus has been the question: What mathematical reasoning, insight, understanding, and skills are required for a person to teach mathematics? Many have worked to develop measures to address this question, most notably Ball and colleagues (Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). In their work they have defined three types of PCK: knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum. Their perspective is that thinking and decision-making in teaching requires integrating knowledge from each of three mathematically rich contexts (content and curriculum, content and teaching, content and students).

Many challenges in measuring PCK when it is framed in this way have been reported (Hill, Ball, & Schilling, 2008). Most test development is for K-8 teaching and includes some algebra and little in the way of advanced mathematics and its semiotics, such as are found in college mathematics. For the purposes of this research, we use an expanded model of PCK. Based on the work of Ball and colleagues, it includes attention to the mathematical communication that emerges in advanced mathematical thinking, including algebra and proof-based mathematics.

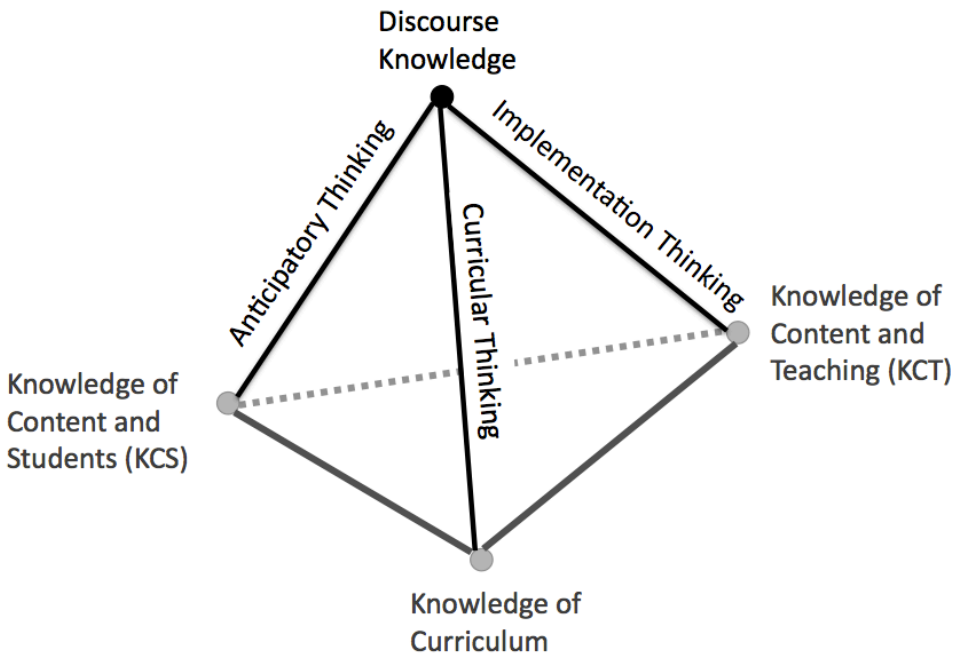

Figure 1. Tetrahedron to visualize relationship among PCK model components. Corners of the base are the aspects of PCK articulated in Hill, Ball, and Schilling (2008).

Working from the foundational three components proposed by Ball et al., the model adds a fourth node of knowledge needed for teaching, discourse knowledge (this aspect brings to the modeling of PCK the mathematical semiotics that was part of Shulman's original description). One way of visualizing the model is as a tetrahedron whose base is the model of Ball et al., with apex of discourse knowledge (see Figure 1). Our attention in building this extended theory has focused on discourse knowledge and the three "edges" connecting it to the components in the Ball et al. model (Hauk, Jackson, & Noblet, 2010; Hauk, Toney, Jackson, Nair, & Tsay, 2013). To situate the results, these four aspects are summarized here.

Discourse knowledge (**DK**) is knowledge about the culturally embedded nature of inquiry and forms of communication in mathematics (both in and out of educational settings). This collection of ways of knowing includes syntactic knowledge, "knowledge of how to conduct inquiry in the discipline" (Grossman, Wilson, & Shulman, 1989, p. 29).

Curricular Content Thinking (**CCT**) is the strategies and approaches to using substantive knowledge about topics, procedures, and concepts along with a comprehension of the relationships among them and conventions for reading, writing, and speaking them in school curricula. In its most robust form, this part of PCK contributes to what Ma (1999) called "profound understanding of mathematics" (p. 120).

Anticipatory Thinking (**AT**) is noticing and strategizing about the diverse ways in which learners may engage with content, processes, and concepts. Part of anticipatory growth involves "decentering" – building skill in shifting from an ego-centric to an ego-relative view for seeing and communicating about a mathematical idea or way of thinking from the perspective of another (e.g., eliciting, noticing, and responding to student thinking).

Implementation Thinking (**IT**) is ways of thinking about how to enact in the classroom the decisions informed by knowledge of content and teaching along with discourse understandings. This includes adaptive, in-the-moment, shifting according to curricular and socio-cultural contexts.

This paper describes the research team's efforts to gather evidence of PCK focusing on this four-part framework. Data came through written assessments and classroom observations of practice. We report here on our progress to date in addressing the following research questions:

(1) Do teacher-participants differ on PCK as measured by the observation instrument pre-program to post-program?
(2) Do teacher-participants differ on PCK as measured by a written assessment pre-program to post-program?
(3) What is the nature of the relationship between pedagogical content knowledge as demonstrated theoretically on the written assessment and in practice on the observation instrument?

We readily acknowledge the limitations of this non-experimental, small-*n* study. As part of the larger body of work to build theory and associated measurements for PCK, the work reported here is valuable in building a foundation. Note that the intention is not to make causal claims. Rather, we are in the early work of testing predictive validity for instruments and exploring potential avenues for capturing PCK and documenting its change.

Methods

Setting and Participants

The setting was a blended face-to-face and online delivered master's degree program in mathematics for in-service secondary teachers. Designed to reach urban, suburban, and isolated teachers in rural areas, the program is conducted using a variety of technologies (e.g., *Collaborate* for synchronous class meetings, *Edmodo* for asynchronous communication). Offered through a joint effort at the Universities of Northern Colorado and Wyoming, cohorts of 16 to 20 new students each year complete a 2-year master's program in mathematics with an emphasis in teaching (about half of course credits in mathematics, half in mathematics education). Cohort 1 participants teach grades 7 through 12 at schools scattered across the two states (see map, right). The program employs both online and hybrid instruction. Teacher-participants take a combination of face-to-face, hybrid two-location site-to-site, as well as synchronous online courses. In the 30 credit-hour program, 18 credits are in the foundations of secondary mathematics (e.g., modern geometry, continuous mathematics), 6 credit-hours are in mathematics education, including a course developed by the Math TLC project called *Culture in the Math Classroom* (Bartell, Novak & Parker, in preparation), and 6 credits are research-focused (a survey of research in mathematics education and an action research thesis project). Of the 16 teachers who started in the first cohort, 14 completed the coursework of the master's program. All 14 completed the pre- and post-program PCK written test. Though we had pre-program observations for all teachers, for this report we had post-observations for 10 of the teachers.

Instruments

The PCK Assessment included released items from the Learning Mathematics for Teaching instrument (LMT; Ball et al., 2008), new items with more complex mathematical ideas modeled on the LMT items and some *Praxis* items. All of these were limited option "multiple choice" items. For some of these limited option items we added open-ended extensions. Multi-year development of the PCK written test included cognitive interviews with in-service teachers and mathematics teacher educators as they completed individual items or several constellations of items (Hauk, Jackson, & Noblet, 2010). The research team created an alignment of the four PCK constructs across items. For example, one item may be identified as presenting both curricular content and discourse knowledge challenges, while another may foreground curricular and anticipatory thinking). These "loadings" of multiple PCK constructs to items was a purposeful part of the non-linear model underpinning the test design. Each item on the written test loaded on at least two of the four PCK constructs. Consequently, factor analysis was not appropriate given this confounding of variables. In addition to the established face validity of the tests, we conducted tests of the constructs' internal consistency (Cronbach's alpha).

The PCK pre-test showed good reliability overall (α=.81), with good reliability on Curricular thinking (α=.81), acceptable reliability on Discourse knowledge (α=.76), and marginal reliability on Anticipatory thinking (α=.55) (George & Mallery, 2003). While the PCK post-test had acceptable reliability overall (α=.75), we did not see at least marginal reliability on the anticipatory thinking (AT) item set. Because of the variable reliability on the anticipatory thinking construct on the written PCK tests, we did not conduct analyses on it. The observation instrument, based on the LMT video observation protocol (see LMT website; development reported elsewhere) showed good reliability overall (α=.85); including good reliability on CCT (α=.84), DK (α=.89), and IT (α=.85); and acceptable reliability on AT (α=.78).

Data Collection

The research team conducted pre-program observations in teacher-participant classrooms in the spring semester prior to teachers entering their first course of the master's program. The team conducted post-program observations in the spring semester three years later; this was two semesters after teachers completed the program. The post-observation data are from a year after completion of the program to give teacher practice time to settle (and avoid detection of an implementation effect that may not be sustained). For both sets of observations, the team observed teachers for three consecutive class meetings (the same researcher(s) visited across the meetings). Like the LMT video protocol, the observation tool used interval recording (Gall, Gall, & Borg, 2007) of 6 minutes each: 3 minutes observed, 3 minutes to record the observation by identifying the presence or absence of each protocol category in the observed segment on the protocol form and to record associated field notes. Each class visit had 7 to 12 segments. Experienced observers trained new observers to use the instrument; new raters practiced using the protocol on video data, conducted their first observations of teachers in tandem with an experienced observer, and team members met to calibrate ratings and reconcile disagreements. Inter-rater reliabilities were greater than 0.8 at each calibration check.

Teacher-participants completed the written pre-test at the beginning of their first class session in the program. Of the 14 teacher-participants who completed the program, 9 completed the post-program written assessment at the program closure meeting. For the 5 unable to attend the meeting, members of the research team administered the test at the teachers' school of employment. Members of the research team created answer keys for multiple choice items and a scoring rubric for short answer items. The rubrics were informed both by expected or desired responses created by item developers as well as cognitive interview data. The procedure for developing the rubric was (1) write a desired response, (2) list other anticipated responses, (3) read the responses from a subsample of participants, (4) come to consensus on a scoring rubric. Two or more research team members scored tests separately, compared scores, and met to negotiate and reconcile any disagreements.

Data Analysis

To date the research team has observed 10 teachers after completion of the program. The counts for each of the variables were summed and divided by the number of segments observed to report a relative frequency for each variable for each teacher. A teacher having a score of 23.25 on “Explicit Talk about Math” means that the rater(s) observed the teacher exhibit explicit talk about mathematics during 23.25% of the segments observed. On both the written assessments and the observation instrument, researchers calculated percent scores for each construct by summing teacher scores on items coded for the construct and taking the percent out of total points possible on each construct. To answer the research question of the impact of the Math TLC master’s program on teachers’ pedagogical content knowledge, we compared entrance and exit data from the PCK assessment and the PCK observations using paired-samples *t*-tests.

To model the relationship between teachers’ PCK during practice (as measured by the observations) and teachers’ PCK during reflection (as measured by the written tests), we conducted a path analysis on the PCK constructs. The model considered the pre-test and pre-observation scores as exogenous variables. Taking the assumption that change in knowledge leads to change in action, the model examined the effects of the exogenous variables (pre-scores) on the written post-test of PCK, then examined the effects of those three variables on the post-observation scores. The path analysis is for Curricular content thinking and Discourse knowledge, since the reliability of the Anticipatory construct was not sufficient and written assessment did not measure Implementation.

Results

PCK Written Assessment

Table 1 presents the results from paired samples *t*-tests on teachers’ percent scores on the constructs of Curricular content thinking and Discourse knowledge for the PCK tests. Teachers’ scores on items coded as Discourse knowledge (DK) increased significantly (t=2.189, p=.047) from the pre-test (M=56.82, SD=15.43) to the post-test (M=66.22, SD=19.09).

Table 1. Paired Samples t-*tests for PCK test Cohort 1. Values are percentages.*

	Pre-program (*N*=14)		Post-program (*N*=14)			
PCK Construct	*M*	*SD*	*M*	*SD*	*t*	*p*
Curricular content	60.93	14.57	65.48	16.74	1.261	.229
Discourse	56.82	15.43	66.22	19.09	2.189	.047

Observations

Tables 2 and 3 present the comparisons of pre-program and post-program observations for the 10 teachers for whom complete data are available. Each table presents the means, standard deviations, and the results of a paired samples *t*-test on each observed variable. Table 2 gives differences on the observation categories aggregated into the four PCK constructs. Table 3 unpacks the information in Table 2 and presents the differences between pre-program and post-program on each item that made up the observation instrument. Because of the number of statistical analyses performed, a cutoff *p* value of 0.0015 (rather than 0.05) is appropriate, based on a Bonferroni correction (Bland & Altman, 1995).

The results in Table 2 indicate increases approaching significance in two constructs: an increase in score for Curricular content thinking (*t*=4.31, *p*=.002) from pre-program (*M*=45.12, *SD*=13.18) to post-program (*M*=56.64, *SD*=1.066); and an increase in Discourse knowledge (*t*=3.92, *p*=.004) from pre-program (*M*=48.25, *SD*=13.47) to post-program (*M*=61.27, *SD*=10.27). Scores in both Implementation and Anticipatory thinking increased, but the difference was not significant at the 0.0015 level.

Table 2. Paired samples t*-tests for PCK Constructs from Observation Instrument*

	Pre-program (*N*=10)		Post-program (*N*=10)			
PCK Construct	*M*	*SD*	*M*	*SD*	*t*	*p*
Curricular content	45.12	13.18	56.64	10.66	4.31	.002@
Discourse	48.25	13.47	61.27	10.27	3.92	.004@
Anticipatory	44.18	12.97	54.26	17.56	1.95	.083
Implentation	59.16	15.13	66.12	9.97	1.54	.159

@ approaching significance, *p* < .015; * significant, *p* < .0015

In all tables, all values increased, indicating some pedagogical effectiveness of the master's program. In Table 3, with the adjusted threshold for alpha, there are two statistically significant results. One was in the observation category "General language for expressing mathematical ideas (overall care and precision with language)." While careful use of general language was seen, on average, in about 49% of pre-program classroom segments, by the end of the program it was present in more than 80% (*M*=80.34, *SD*=19.71). The other significant result was in the category "Mathematical descriptions (of steps)" (i.e., segments where the teacher or students accurately used explicit language to describe the steps of some mathematical process). On average, across pre-program observations, this was seen in about 40% of class segments (*M*=40.28, *SD*=21.94), increasing to almost 70% of the time, post program (*M*=68.10, *SD*=19.31). Three other observed variables appear to be approaching significance (i.e., *p*<.01): the percent of segments where (a) student voices were present in the room (increasing from 80% to 90% of segments), (b) teachers were observed to use conventional notation (increasing from 54% to 90% of segments), and (c) fewer mathematical errors occurred (decreasing from about 4% of the time to nearly 0%).

Table 3. Paired Samples t*-tests for Observation Protocol Variables.*

	Pre-program (*N*=10)		Post-program (*N*=10)			
Observation Item	*M*	*SD*	*M*	*SD*	*t*	*p*
Format for Segment						
Whole Group	51.61	22.16	63.39	18.40	1.170	.272
Small Group	23.79	22.51	39.26	22.51	2.832	.020
Individual	41.70	26.03	28.98	12.15	-1.610	.142
Lesson/Segment Type						
Review	26.46	18.03	22.46	14.17	-0.549	.596
Introducing tasks	7.23	4.74	10.64	5.31	2.262	.050
Student work time	45.00	24.16	50.04	16.76	0.586	.572
Direct instruction	24.15	15.27	33.00	16.53	1.201	.260
Synthesis or closure	5.77	4.91	8.10	6.02	1.147	.281

Table 3-Continued. Paired Samples t*-tests for Observation Variables.*

	Pre-program (*N*=10)		Post-program (*N*=10)			
Observation Item	*M*	*SD*	*M*	*SD*	*t*	*p*
Math Teaching Practices						
Voices – Students	79.82	18.32	89.29	15.88	3.375	.008@
Voices – Teacher	80.77	21.98	93.81	8.23	1.949	.083
Real-world Problems	26.55	28.47	36.50	32.44	.826	.430
Interprets Students' Work	63.33	18.39	73.01	13.75	2.120	.063
Explicit about Tasks	82.20	16.20	87.52	12.42	.916	.384
Explicit Talk about Math	59.03	27.18	75.59	11.91	1.801	.105
Explicit Talk about Reasoning	29.93	23.18	49.48	17.94	2.821	.020
Instruction Time	86.10	10.20	87.02	6.81	.249	.809
Encourages Competencies	67.07	26.83	45.04	40.52	-1.420	.189
Knowledge of Math Terrain						
Conventional Notation	54.39	21.38	79.95	15.25	3.353	.008@
Technical Language	72.59	17.38	77.67	13.53	.760	.467
General Language	49.06	13.88	80.34	19.71	4.528	.001*
Selection for Ideas	87.17	8.31	91.16	5.68	1.989	.078
Selection to Represent Ideas	31.70	23.97	43.64	25.51	1.892	.091
Multiple Models	17.80	14.90	33.69	24.20	2.138	.061
Records Work	59.67	28.25	52.01	20.20	-.585	.573
Math Descriptions	40.28	21.94	68.10	19.31	5.003	.001*
Math Explanations	40.65	23.26	55.80	16.29	1.782	.108
Math Justification	14.32	16.13	23.09	11.05	1.928	.086
Math Development	84.50	16.57	88.67	6.11	.753	.471
Errors – Not Present	96.27	2.67	99.78	0.69	3.858	.004@

@ $p < .015$, * $p < .0015$

Relationship between Observation and Written Assessment

The figures below display the results of the path analyses exploring the effects of the program's pre-scores on the post-assessment and post-observation scores. Figure 2 shows the full model for Curricular content thinking (CCT). There was a significant effect of the Pre-Test (β=.88, *SE*=.17, $p < .01$) and no significant effect of the Pre-Observation on the Post-Test of CCT. There was a significant effect of the Pre-Observation (β=.68, *SE*=.17, $p < .05$) on the CCT Post-Observation. There was no significant effect of the Pre-Test on the CCT Post-Observation, but interestingly the effect was negative (β=-.60, *SE*=.27). Although the effect of the Post-Test on the Post-Observation was relatively high (β=.81, *SE*=.28), it was not significant, which may be due to the small *n* and large standard error.

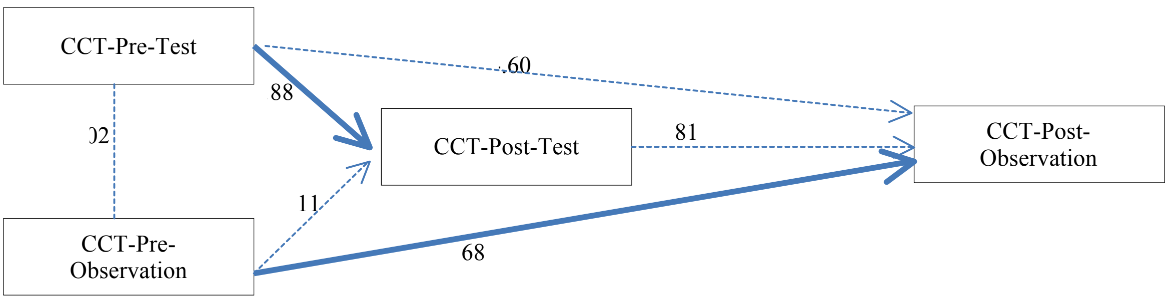

Figure 2. Path diagram for CCT construct.

Figure 3 shows the full model for Discourse knowledge (DK). There was a significant effect of the Pre-Test (β=.78, *SE*=.26, $p<.05$) and no significant effect of the Pre-Observation on the Discourse Post-Test score. There was no significant effect of either the Pre-Observation or the Pre-Test on the Post-Observation of DK, although, like CCT, the effect of the Pre-Test was negative (β=-.58, *SE*=.19). Finally, there was a significant effect of the Post-Test on the DK Post-Observation (β=.92, *SE*=.17, $p < .05$).

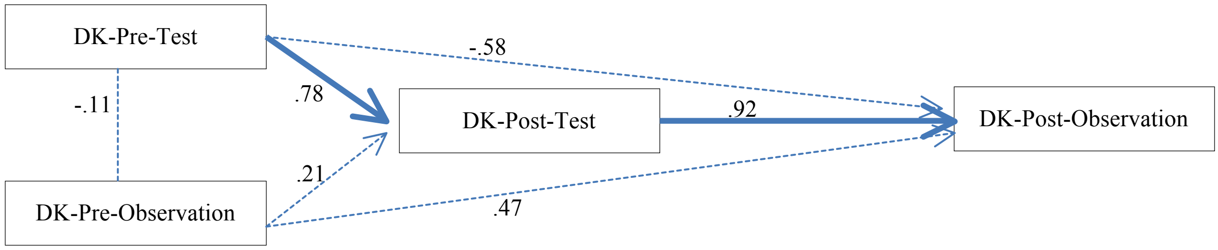

Figure 3. Path diagram for DK Construct.

Discussion

Because of the small sample size, the study is underpowered for full validation of the assessment and observation scores. Additionally, the small sample size makes generalizing the results problematic. What is apparent is that pre- to post-program written test score changes suggested positive potential outcomes of the master's program in the target area of investigation: development of pedagogical expertise for teaching secondary mathematics, particularly in the communication skills of responsive classroom discourse. The significant increase in curricular content thinking (CCT) from pre- to post-program teacher observations may reflect the master's program emphasis on increasing participant understanding of advanced mathematics and deepening secondary-school-level-appropriate conceptual connections. This is evident in some of the significant increases on individual variables in observation categories. For example, the significant increase in the use of conventional notation may indicate that the master's program supported teacher-participants in the habit of using conventional notation to communicate. In addition, the mathematics courses required participants to be explicit about their thinking, reasoning, and justification of answers, which may help explain the significant increase in mathematical descriptions category. However, there were only small (non-significant) increases in the mathematical explanations or the mathematical justification of the reasoning process, so more work needs to be done in the program to support teachers' attention in these important realms of mathematics teaching and learning (perhaps as they challenge the prescribed curricula, which tend *not* to foreground these things). Finally, the reduction in observed errors may indicate a stronger content knowledge for teaching secondary mathematics.

The significant increase in Discourse knowledge (DK) on the written test and in observations may be related to the master's program mathematics education courses. In particular, these courses employed and examined reports of responsive mathematics pedagogy. That is, they made explicit use of the research-base on student-centered classrooms and the mathematics instructional practices that support students in the construction of knowledge (rather than the transmission of knowledge by teachers). For example, observers saw significant increases in the percent of small group work and in students' voices in the classroom. This may indicate that teachers' practices shifted to more decentered (or some forms of "learner-centered") approaches. Additionally, the program included several credit hours of reading and writing about mathematics education research focused on the NCTM process standards. The increase in discourse knowledge in general may be attributed partly to the pedagogy courses that allowed participants to read research and experience what good mathematics discourse "sounds like and feels like" (Cohort 1 participant, personal interview, October 8, 2012). Finally, the *Culture in the Math Classroom* course experience included several culture and discourse-specific awareness building activities, scaffolding teachers in decentering their instruction. We suspect that this course and the potential shift in perspectives that teachers may have gained from it will turn out to be a significant predictor of change in Discourse knowledge in the program – part of our ongoing work investigates this hypothesis.

The path analyses relating PCK as demonstrated on the written test and in practice provide interesting results that need further investigation. As noted, the path diagram for discourse knowledge, Figure 2, suggests that the written test may have predictive value in capturing classroom practice. If this turns out to be a robust result, across populations of teachers, it could reduce or eliminate the need for expensive classroom visits when attempting to determine impact on practice (e.g., it may be the pre-program observations and pre/post written tests can give the impact information without the need to re-visit classrooms post-program). Researchers need to conduct further investigation into the ways to measure these constructs and to extend the research to larger, more generalizable samples to verify these results. Additionally, with further data we hope to clarify the degree to which the negative, albeit not significant, correlation of the pre-test with the post-observation is an artifact of small n and within-sample variability or may be an indicator of another variable, possibly related to intercultural orientation and the relative impact of the *Culture in the Math Classroom* course on that orientation.

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