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Culturally Responsive College Level Mathematics

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The goal of this chapter is to describe what it might mean for college level mathematics teaching to be culturally responsive and illustrate how culturally responsive collegiate mathematics teaching and learning can look. Our focus is on effective college mathematics instruction for non-mathematics majors in service courses like calculus and liberal arts mathematics. Culturally responsive courses in the mathematics major are possible, but require a more extensive discussion about the specific nature and purpose of the mathematics major within a department before change is possible.

After providing some background, we offer common views of college mathematics teaching in the overlapping contexts of academic, workforce, and social justice concerns. Secondly, we give several short examples from the perspective of college professors about the nature of their instructional practices, including cultural responsiveness. Thirdly, we address the nature of culture and the repertoires college students and instructors build – of ways of seeing, communicating about, and engaging with these concerns. Fourthly, we provide two detailed examples of culturally responsive teaching and curricula in courses that currently exist along with some of the successes documented in these courses. We close with suggestions for how to improve the educational environment for students and instructors through the tenets of culturally responsive pedagogy. Throughout, we connect our observations with existing critical educational theories. That is, we employ common *academic mathematics cultural* practices: we start with some background information and several motivating examples, give some definitions (after already having used key terms in context), provide two extended examples, making connections along the way, and conclude with a summary of what we think these all show.

Background

For most U.S. students entering college, of all races, classes, ethnicities, and home language groups, mathematics means "computation" and mathematics beyond arithmetic is seen as having little relevance to everyday life (Hauk, 2005; Leder, Penhkonen, & Torner, 2003; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). In fact, mathematics "is commonly perceived as the antithesis of human activity - mechanical, detached, emotionless, value-free, morally neutral" (Mukhopadhyay & Greer, 2001). Nonetheless, like the other authors in this volume, we assert that mathematics *is* a human activity and is value-laden and culturally informed. Any human endeavor, including mathematics, that has an associated set of values (e.g., elegance), preferred ways of communicating (e.g., proof), and rules for inclusion (e.g., logical validity) is culturally in Drive, not Neutral (Davis, Hersch, & Marchisotto, 2003; Ernest, 1998)

In our view, teaching and learning in college mathematics involves managing the tensions among at least four significant factors, the demands of:

(*a*) *Academic mathematics* culture for (re)producing mathematics in ways authentic to the traditions from which it arose,

(*b*) *Society and the state* to produce mathematically competent workers,

(*c*) *Global ecology and humanity* to be critical thinkers in our use of mathematics, and

(*d*) *Multiple student communities* for mathematics teaching and learning.

These are akin to Gutstein's (2007) *classical* (he groups academic mathematics and societal demands under this one heading), *critical*, and *community* knowledge aspects of teaching mathematics. Each of the four can be seen, a la Bourdieu, as a *field*, with associated *habitus* and *relational structures* (Grenfell & James, 1998). College mathematics instructors operate at the nested intersection of these fields, where *a* is nested inside of *b*, which is in turn nested in *c* while *d* may or may not overlap *a*, *b*, or *c*. The *relational structures* at work in assuming and asserting power as an instructor in these fields depends on many factors, including where an instructor is teaching (e.g., 2-year community college or doctoral intensive research university) and what courses an instructor is teaching (e.g. major or non-major courses). The challenges of moving into culturally responsive college mathematics teaching for an instructor at a vocational-goals oriented community college may be relationally and structurally different from those faced by one at a community college where the primary goal is feeding students to the local university and different again in other ways from the challenges a university mathematics professor might face. However, Bourdieu's attention to the *habitus* of people in the field – "the systems of dispositions they have acquired by internalizing a determinate type of social and economic condition" (Grenfell & James, 1998, p.169) provides a mechanism for analysis that works across these different contexts. College mathematics instructors and students deal every day with the pulls of the intersecting fields and the sometimes dissonant aspects of habitus associated with each. Some instructors focus on one of the fields at a time: for example, looking at rich development of the classical or a sole focus on the societal in their instruction. Some have pledged allegiance to the classical, saying:

> Our primary responsibility as mathematicians is not to students, but to mathematics to preserve, create, and enhance good mathematics and to protect the subject for future generations (Palmer, 1997, p. 10).

In particular, the classical habitus has it that those things that become "non-mathematical" are excised from mathematics, including applications:

> Over the centuries, mathematics has outsourced many (usually applied) sub-domains when they developed their own ways of thinking and working (cf. Laugwitz, 1972). By considering them not to be a part of mathematics anymore, inconsistencies or conflicts could be removed in an easy way. Even today, there are disciplines of mathematics (like scientific computing or other parts of experimental mathematics) whose standards have been removed from the widely accepted mathematical standards. (Prediger, 2002, p. 8)

By contrast, pledging allegiance to *b*, the societal, means the instructional goal is one of shaping students who can apply mathematics (rather than shaping conservators of mathematics). Among the societal forces at work in universities in the U.S. today is the push to use "business management" styles in academe. This has been felt as people from commerce become university administrators and through the less obvious but more powerful pressure exerted through the profit-based views of members of university Boards of Trustees. Additionally, state funding of public higher education means that demands felt by the state are passed on to university administration and thence on to faculty (e.g., by businesses who position themselves as customers of the state, with the products being college graduates).

Some instructors do a balancing act, taking the classic academic and societal as both important. And some, who move in the direction of culturally responsive instruction, take *c*, globally contextualized critical knowledge, as the driving force in making instructional decisions. That is, some instructors design college teaching for critical understanding while being responsive to the demands of the classical and societal as means to the end of critical engagement with mathematics. As Gutstein (2007) noted, "it is often the case that community knowledge is already critical, but context matters" (p. 111). One of the basic tenets of our approach to culturally responsive pedagogy is in using *d*, community knowledge, as a foundation for working with college students to pose and solve problems while developing both classical and global critical knowledge.

Given that the traditions in Western education call for a knowledge of abstract concepts in mathematics and given that two out of every three new jobs in the U.S. require some post-secondary education in broad and flexible critical thinking ability (Carnevale & Desrochers, 2003), how do we support the next generation of U.S. students as thinkers, workers, and global citizens? Colleges and universities must prepare culturally competent graduates who are aware of and skilled in moving among multiple social, cultural, and linguistic contexts (Middlehurst & Woodfield, 2006). Among the challenges in shaping collegiate mathematics instruction to meet quantitative literacy and cultural competence goals are the inertia of academic mathematics culture, the assimilationist underpinnings of the majority society (or as Delpit (1996) says, the *power culture*), and the very slow diversification of college mathematics faculties.

The slightly greater socio-cultural diversity and difference in gender balance for college mathematics faculties from that in schools – see Tables 1 and 2 (U.S. Census Bureau, 2000; National Center for Education Statistics, 2006) – may mean that the college level instructional environment is a more fertile field for cultural responsiveness to grow in at least two ways. First, many college mathematics faculty are already interacting with colleagues who are of some "other" home culture yet who are also participants in the academic mathematics culture, that is, with colleagues who have a "dual status frame of reference" (Ogbu & Simons, 1998, p. 156). Moreover, college students may encounter, in addition to the disconnect in moving from "school" to "college," a cultural conflict when they find themselves with an instructor who is seen as "other." This otherness might come from the perception by a student that a mathematics instructor is an alien being or might come from perceived socio-cultural, gender, or linguistic community differences.

Table 1. *Diversity of Elementary, Secondary, and Post-Secondary Faculties in the United States.*[a]

Category [b]	Elementary	Secondary	Post-Secondary
All others (e.g., 2 or more categories), non-Hispanic	0.5%	0.5%	1.0%
American Indian or Alaskan Native, non-Hispanic	0.5%	0.5%	0.5%
Asian, non-Hispanic	1.5%	1.5%	8.0%
Hispanic	5.5%	5.0%	4.5%
Black, non-Hispanic	9.0%	6.2%	6.0%
White, non-Hispanic	82.5%	86.0%	80.0%

a. Data are rounded to nearest tenth from 2006 projections based on U.S. Census (2000).
b. Group "race" labels are those used in the U.S. Census (2000).

Table 2. *Gender of Elementary, Secondary Mathematics, and Post-Secondary Mathematics Faculties in the United States.*[a]

Category	Elementary	Secondary Mathematics	Post-Secondary Mathematics
Men	21.0%	45.1%	75.3%
Women	79.0%	54.9%	24.7%

a. Data are rounded to nearest tenth from 2006 projections based on U.S. Census (2000).

Moreover, this cultural disconnect happens even for the academically well-acculturated, in the mathematics graduate school experience (Herzig, 2002, 2004).

Before we move on, we position ourselves as authors, mathematicians, and college teachers. All the authors are PhDs in mathematics and each of us has taught college mathematics for more than 10 years. Within our group we have also taught elementary, middle, and high school. We have taught in the U.S. and in Africa. We have conducted basic and applied research in mathematics (logic, group theory, number theory, dynamical systems, climate modeling) and have done basic and applied research in mathematics education. Each of us has taken a different route in coming to our interest in culturally responsive mathematics education: one coming from the U.S. majority culture and going into teaching in very diverse and new settings, one coming from experiences rich in dealing personally with mathematical and societal racism, and one through her experiences with manifestations of sexism. Our agenda is promoting the opening of our views of instruction to include the critical perspective as we develop ourselves and as we help our students develop mathematically.

We frame the rest of our presentation by describing next the four dominant paradigms in college mathematics instruction, including their connections to classical, societal, and critical allegiances and ways these can be infused with community knowledge in culturally responsive pedagogical approaches. As promised, we will start with some examples.

The Instructor Speaks: A Collection of Short Examples

Current views of instruction fall into four broad categories (Grundy, 1987): *transmission, product, process,* and *praxis*. Below, the comments from college professors are exemplars crafted by the authors to condense actual interview responses. These come from our ongoing collegiate mathematics education research projects. It should be noted that the authors are not arguing a value-laden hierarchy to the models as we have presented them below. People come to college mathematics instruction with differing funds of knowledge, differing world views, and a variety of approaches to instructional change. Our contention is that learning in the context of any one of the four instructional paradigms can be made more effective by expanding instructional design to include responsiveness to community knowledge. The four models are ordered from most common to least common in collegiate mathematics instruction (according to our review of the literature and our own research).

Transmission Model

At the college level in this mathematics instructional approach, curriculum is the content of the syllabus and textbook. Students are vessels to receive this content and they are responsible for structuring it for their own future use as thinkers. Instruction is the act of speaking (transmitting) the content. Students and teachers are guided by the demands and constraints of academe (classical forces). Assessment is whatever the habitus and norms in the mathematics department suggest (most often a midterm and a final exam). The transmission model has a large and stable following at the college level and can be exemplified by the comment of Professor T: “My job is to present the information from the book to the students as clearly as possible.” The traditional transmission model assumes that what is “clear” to students has already been determined by what the teacher or textbook says is important. This approach to instruction is common for lower-division non-major and mathematics major courses. In his student community responsive version of this curricular approach Prof. T. goes on to say:

> *How* I present things depends on what does the best job of being clear to the students. When I have a lot of students from the city, I can ask them to think about financial implications for the calculus we do, but when I have a lot of kids from the farms, I change my examples so that they think about how managing water resources on a ranch can be modeled by things like difference equations. If the class is mixed, I spend a little time on each example.

Though an instructor with a transmission approach typically uses lecture as the dominant form of instruction, this is not the sole method. For example, separate computer lab sessions with very directed and structured lab activities may be added to a course. In addition to lecturing, Professor T used *Mathematica* software documents, called notebooks, where students clicked on entries to reveal computations or typed in formulas as evidence of mastery of procedures. This instructional approach was based on, as noted by Professor T, the “assumption that if a student practices a procedure enough then [conceptual] understanding will follow.”

Product Model

In this instructional approach, curriculum is a set of goals about mathematical knowledge acquisition along with assessable objectives and definitions for what constitutes evidence of learning. Students are the raw material to be shaped by instruction into a certain product: the educated worker. Instruction is the calibration of presentation and assessment that results in that quality-assured end product of the college graduate as worker. Students and teacher are guided by the demands and constraints of a capital economy (societal forces). Many of the courses for mathematics majors are taught with this approach. The traditional *product* view can be illustrated by the comment of Professor I:

> The way I see it, I should present to students what they need, cover all the material, so they can solve the problems they see in the book and on the test. My goal is to prepare students for the next class...even if they say this is the last math class they're ever going to take, there is always a next class and that's what I'm getting them ready for. Some students just won't get it, usually it's the ones who shouldn't have been allowed in, to the college, in the first place, but that's the way it is.

The product view can also be seen at the college level in the assessment strategy often used in large, multi-section, coordinated mathematics service courses. The same (or similar) exams are administered in all sections of the course and the tests are graded uniformly using a common rubric (or a common grader; e.g., if there are 10 instructors for 500 students in a calculus course and 10 items on the exam, Instructor X grades Item 1 on all 500 exams, Instructor Y grades Item 2 on all 500 exams, etc.). Instruction and assessment follow the assembly-line model. A community responsive product-based instructor will modify presentation and assessments based on students, like Professor B:

> For example, students need to master the idea of slope and connect it to rate of change and the derivative. So, when I teach calculus, I give students graphs of distance versus time and I make up a story that goes with it from my experiences as a hiker. Then I ask them to create their own stories about that graph and three others for homework. One or two of the stories they make up show up on a quiz later in the week: students have to work backwards, draw the graph from the story. This builds their understanding of the relationship between slope and rate of change. Because the students made up the stories, the math is connected to their lives; and because we share some stories, we get to know each other better.

In fact, the nature and use of word problems as a responsive tool in college mathematics curriculum is a fertile ground for research. As Gerofsky (2004) noted in her exploration of word problems as genre, the habitus in academic mathematics is to consider the kinds of word problems found in most college textbooks as a way to "give the student time to think" (p. 73). And, many mathematicians who teach college service courses might agree with the university professor who told Gerofsky that the contexts in word problems do not matter; that, in fact, they tend to ignore the stories – even if "horrifying" (p. 71) in content when considered carefully. This is because the purpose of word problems in mathematics is seen as stripping away the words: "a word problem is a model of a real problem and then mathematics is a model of the modeled problem" (p. 70).

Also common to the product-based model is the use of computer labs and projects. In a product-based approach, technology use may depend on how much the instructor anticipates students needing computerized mathematics skills. If the mathematics course is for engineers, computer use may be much more likely than if the course is for "pure mathematics" students. In a culturally responsive product-based approach, especially in college mathematics courses at the level of calculus and below, computer use may be more focused on worker skills like using spreadsheets or finding and distilling information from the world wide web rather than using computationally powerful software.

Process Model

From this perspective, curriculum is the process of developing thinking skills for "dealing with the world as it is" – each classroom full of students may learn a different collection of content, but all are being shaped for citizenship within the *status quo* of majority society (classical and societal forces in tension). Students are sense-making participants in the development of their own understandings. Instruction is the act of pacing and facilitating learning for the particular group of students in the room. Students and teacher are guided by the demands and constraints of the larger society. The *process* view, as it is often realized in traditional college mathematics instruction, is represented in the comments of Professor J:

> I see my responsibility as making sure students can think, can be flexible and use mathematics. They should know more than the facts, they should be able to solve problems they have never seen before. Okay, often they can't, but it's what I aim for. I cover as much as I can. It's why I have those projects the students complain about, to help them learn how to work together and think on their own.

Professor P agreed with Professor J and was working to expand on the ideas in the direction of cultural responsiveness, saying:

> Cultural responsiveness is about meeting the needs of the individual students as we, the instructors, understand that. Although it would be useful to know more about the cultural context they come from, we can be responsive to the individuals and teach them math without knowing the

> details of their lives. Those details come through in the project topics they pick, they show up in what the students want to work on.

That is, Professor P was open to the ideas of community responsiveness and was still working to build an understanding of socio-cultural experience and of its centrality to the mathematical experience of his students (e.g., still coming to an awareness of "how the negotiation of mathematical norms in the classroom mimics and reproduces the larger social relations that exist outside of the mathematics classroom (i.e., some [students] are shut out of the process)." Martin, 2006, p. 204). The efforts of Professor P to move from a traditional product model to a culturally responsive form of the process view are detailed below in the first of our two extended examples, about teaching Applied Calculus.

Praxis Model

Within the praxis instructional paradigm, curriculum is the collective practice of teacher and students engaging with and shaping the world through knowledge of mathematics and other content. Students are knowledge-generating participants who apply their experiences of the world and understandings of mathematics to analyze and influence the world around them. Instruction is the act of supporting students in critical discourse, planning, and implementation of ideas. Students and teachers explore, challenge, and redefine the demands and constraints of multiple stakeholders in local and global communities. This view is associated with the theory of *critical pedagogy* (Freire, 1970). The *praxis* view can be seen in the comments of Professor D:

> For me, *teaching* and *learning* are not "two sides of the same coin" so much as two faces of a mirrored tetrahedron. The other two faces are *person* and *community*. Well, commun*ities*, since we each belong to many communities. I teach mostly first-year students, mostly European American, and mostly from fairly stable and affluent backgrounds. They are accustomed to looking in the mirrored face I call "person" and seeing themselves. They are used to looking in the mirrored face I call "teaching" and having a Dracula-like experience where they do not see themselves, but they do see the teacher standing behind them.... My goals are for them to learn mathematics to use and define their views of the worlds they live in and to create complete reflections, with lots of background detail, and *including themselves* in each of the four faces: teaching, learning, person, and community.

Though highly consistent with culturally responsive pedagogy, Professor D's view is not especially responsive to the personal variation in experiences her students bring with them to the classroom. Professor T's transmission-based view may actually be more explicitly *responsive* to student community than Professor

D’s. Below, in our second detailed example, we illustrate Professor H’s attempts at culturally responsive praxis-based instruction in Liberal Arts Mathematics.

Before we move to the extended examples alluded to, we feel it is necessary to define some of the terms whose meaning the reader may have gleaned from context. The examples from instructor comments were designed to illustrate the state of college mathematics instruction and to motivate the next section where we discuss cultural repertoire, cultural dissonance, and cultural responsiveness among other things.

Culture and the Classical, Societal, Critical, and Communal

Every human living in proximity to other humans begins enculturation at birth (if not before). In the U.S., even the “culture of no culture” is a culture. For example, the “culture of forgetting” among European Americans that emerged around 1940s Dust Bowl survival had values, norms, and artifacts: forgetting antecedents and moving on were valued as were careful choices about assimilation and avoidance; nuclear family structure was present; there was a norm of non-communication about a painful past; and portable tools (and portable religion) were valued over larger, more cumbersome, physical manifestations of culture (Gregory, 1989). What is quite different for many descendents of this “forgetting” culture from other ethnic-based groups in the U.S. is that the centrality of oral history and kinship, of family memory, may be largely absent. Instead, isolation is the basis of enculturation; perhaps originating in the nihilistic beliefs of the Christian sects who were the forebears of these White migrants. Similar analyses of other cultural groups in the U.S. (e.g., in Grundy, 1987 or Ogbu & Simon, 1998) make it clear that though the details of cultural traits may differ, including those associated with socio-economic status, common categories describing the nature of most cultures exist.

Culture

As with any force to be reckoned with, in order to be responsive to the cultural pressures in college mathematics courses, one must first identify them. For our purposes, *culture* is a collection of learned ways of seeing and interacting with the world and a slowly evolving intergenerational template for the shaping of those learned behaviors. Key aspects of any culture are its:

(*a*) systems of meaning (e.g., semiotics and language);
(*b*) social organization (e.g., community, family, kinship, nation);
(*c*) value structures (e.g., ways of determining and sustaining beliefs);
(*d*) products (e.g., artifacts and tools)

Because mathematics is a human endeavor, it has an associated compound of culture traits. These traits include valued approaches to analyzing, judging, evaluating, interacting with, and mathematizing the world that have informed the development of mathematics as an intellectual enterprise (Hiebert, 2003; Tymoczko, 1998). These classical foundations for what is valued in mathematics and in its teaching and learning inform college mathematics textbook development and saturate the pedagogical assumptions behind the preparation of future college faculty[1] (Center for Education, 2003; Rishel, 2000).

As learners, our community knowledge – including personal life experiences, relationships, and values – may or may not coincide with the knowledge valued in the nationally dominant culture. Consequently, our engagement with learning will be mediated by the consonance (or dissonance) between personal community culture, classical mathematics culture, and large scale societal demands encountered in the classroom (Abreu, Bishop, & Presmeg, 2002; Rodriguez & Kitchen, 2005). Most implementations of the transmission, product, and process models embrace the assimilative view that all students should aspire to the majority culture. Because of this assumption, formal education in mathematics is a socializing experience that only will build smoothly on the informal education gained at home for a learner whose home culture closely resembles the power culture. For the other half of school children, whose home culture may not aspire to middle class European/Anglo American male privilege, school mathematics can become an acculturative challenge requiring students, parents, and teachers to resolve conflict between personal and other cultures (Bishop, 2002; Rodriguez & Kitchen, 2005; Martin, 2006).

Cultural Conflict

Many students who make it to college mathematics courses have developed adaptive skills in moving among the cultures of different fields and negotiating the associated relational structures and aspects of habitus. Nonetheless, discord will naturally arise in any socio-culturally heterogeneous group of humans. The resolution of conflict is a necessary condition for negotiating influence in a socially-mediated milieu, be it a research group, a mathematics department, a classroom, or a conversation. However, since human beings are active decision makers or *agents* in their own cultures, "resolution" does not necessarily mean that one group of ideas is sacrificed to another. Such conflict can be resolved through a consensus-based balancing of cultural demands. When faced with a situation that creates cultural dissonance, people can find ways of managing themselves and their surroundings that may go beyond accepted customs and cultural prescriptions (Bates & Plog, 1980). For example, a

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[1] This may be how the Western academic mathematics culture arose from mathematics in the service of a royalty-driven society, rather than the capitalism-driven model of the state common in the U.S. today. For example, there is no ethical training in becoming a mathematician – potential ethical issues around building weapons were a non-starter when mathematics was developed in the days of monarchies rather than democracies.

Taiwanese immigrant graduate student teaching calculus to a diverse classroom full of U.S. undergraduates faces a different set of negotiated resolutions to linguistic and cultural conflicts than an African American PhD mathematician teaching the "same" course to a classroom of predominantly European American U.S. students. Just as cognitive dissonance and disequilibration have come to be valued as opening cognitive space for the generation of learning (Piaget, 1963), cultural dissonance can pose opportunities for the creation of new repertoires for learning classical, societal, community, and critical mathematics.

Cultural Repertoire

For one exposed to the demands of operating outside of privilege in either a well-known or foreign culture, context is likely to play a very large role in how decision-making happens and in which cultural register or repertoire is relied on for interaction (Even-Zohar, 1997; Swidler, 1986). For example, the mathematical register is something mathematicians know as a privileged collection of ideas and symbols for use in communicating mathematics (Wells, 2003). All teachers have some knowledge of this tool in classical mathematics, but for most (especially grades K through 8 teachers) exposure to, familiarity with, and comfort in using this mathematical register is limited. Even more dependent on socio-cultural community context are students' uses of the mathematical register. For example, mathematics classes may involve "work" for the student and her parent, "activity" for the teacher, and "problem-solving" for a mathematician or mathematics education researcher (Civil, 2002). Weaving together adaptations and negotiations around cultural traits into cultural "repertoires" or "tool kits" for functioning in cultural contexts other than the first-learned is one way human beings negotiate the rapids of cross-cultural social interaction (Swidler, 1986; p. 273).

Learning and Teaching

We hold that learning is the process by which humans construct understanding as individuals and as collectives. As Paul Halmos (1994) put it: teaching "is not to tell students but to ask them, and better yet, to inspire them to ask themselves – make students solve problems, and better yet, train students, by example, encouragement, and generous reinforcement, to construct problems of their own." (p. 851). Students are often their own teachers, separately and severally. Effective teaching requires responding to autonomous as well as communal learning needs. We presume that social and physical realities exist for each individual and for collectives (even as we do not assume we have knowledge of these realities). Such a view makes room for negotiating a social constructivist epistemology based on the two principles of radical constructivism: that knowledge is actively constructed by a person (not passively received, no matter how "passive" the person may outwardly seem) and that "the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality" (von Glasersfeld, 1989, p. 182). Note that the second condition means that the act of cognition is cultural: cognition organizes *experiences* in our physical and social realities (Ernest, this volume).

Culturally Responsive Pedagogy

The phrase "culturally responsive" has acquired many interpretations over the years. For clarity, we describe how we are using the term and illustrate its meaning in college mathematics teaching and learning. For our definition we rely most on Gay's (2000) representation of the idea:

> Culturally responsive pedagogy simultaneously develops, along with academic achievement, social consciousness and critique, cultural affirmation, competence, and exchange; community building and personal connections; individual self-worth and abilities; and an ethic of caring. (p. 43)

Key to cultural responsiveness in curriculum and instruction are a multiculturally- and community-aware definition of learning and the simultaneity of intellectual and interpersonal appropriation and feedback represented by the word "responsive." The basic framework of culturally responsive instruction is *not* about how to get students to change their ways of seeing and interacting with mathematics to align with those of classical academic mathematics or with any other particular culture. Culturally responsive instructional approaches encourage the creation by each individual of *multiple shared repertoires*, in particular, the development of overlapping and mathematically rich cultural repertoires and the skill to identify, choose, and act within and between them.

A culturally responsive college instructor actively models actions and approaches to engaging with mathematics that are culturally aware as well as socially and ethically informed. The socio-culturally heterogeneous nature of the student bodies at the universities where we have experience has shaped our views. For example, helping middle-class college students, European Americans included, to examine privilege and oppression and learn about easily realized potentials for harm (to themselves, to others, and to the world), can be a significant part of the work of culturally responsive pedagogy in college level mathematics. As was indicated in the earlier quotes from different professors, one can hold any of the four instructional views and engage in some form of culturally or community responsive pedagogy. Below, after presenting details of two different extended examples of attempts at culturally responsive college mathematics teaching, we outline six factors of culturally responsive college mathematics instruction.

Extended Example: Bringing Cultural Responsiveness to Applied Calculus

We offer details of a culturally responsive applied calculus course at a large U.S. university (17,000 students) that is part *product* and part *process*. Professor P introduced an alternative to the traditional general education calculus and statistics mathematics sequence at the university. Due to departmental constraints regarding the development of mathematics courses, these alternatives to the traditional applied calculus and introductory statistics courses had to be offered as a sequence with new titles and course numbers and offered through a unit separate from the mathematics department. Both courses, *Environmental Mathematical Modeling* and *Environmental Statistics,* were approved for use by the university as alternatives to the pre-existing courses as prerequisites. The sequence was made up of two, one-quarter (10 week), classes that met four times each week, 50 minutes per session. The curriculum in Professor P's "environmental" courses emphasized communication about and student-generated projects around mathematical ideas in majority culture contexts, particularly those related to ecological models. The "traditional" courses focused on practice with procedures and computational formulas, particularly those related to business models. As a result, the few students from the environmental courses who later went to the next higher level of mathematics found themselves facing different challenges than the students who moved from the traditional courses on to the next level.

The details given below, though specific to the environmental calculus course in a particular Fall term, were representative of the course processes both before and since that term for multiple instructors. Professor P, a PhD mathematician in the mathematics department taught the course considered here with the help of a graduate assistant. The graduate assistant was present in class to work with students, took attendance (a key component of the course grade), and was responsible for reading and giving feedback on student journals.

The environmental calculus course was problem driven. There was a midterm exam (product model), but grades were also based on other activities (process model). The course grade was determined by one midterm exam, weekly working group problem-solving grades, a cooperative final project (individually reported and graded), and a collaborative project poster presentation (group presented and graded). In addition to these graded assignments, each student kept a journal reporting on engagement with mathematical ideas and course processes.[2] To obtain a particular grade, students had to demonstrate performance at a

[2] In the term considered, the only prompt given to students was to write about the day. Subsequent analysis of student journals led to the creation and use in later terms of specific prompts in the areas of mathematical content, course process, and affect or attitude (as suggested by Dougherty, 1996).

minimal level of competence on each assessment and attend class regularly (e.g., missing class more than three times would preclude an A as course grade).

Course Processes

Learning goals articulated on the syllabus involved both content and attitudinal components and reflected the instructor's weaving together the product model goals of creating workers (e.g., ability to work in research teams) and process model goals of preparing students as supporters of the status quo (e.g., "ownership" of mathematical ideas and citizenship skills in reasoning about data). The "performance goals for student progress" from the Environmental Mathematical Modeling course syllabus:

> Students will:
> 1) develop a facility with and ownership of the mathematical concepts of rates of change and accumulation,
> 2) be able to analyze raw data to make reasonable claims using mathematical models of that data,
> 3) be able to communicate mathematical analyses orally and in written form to peers,
> 4) have increased information literacy,
> 5) have increased competence and confidence with respect to the use of mathematics in their lives, and
> 6) have increased competence and confidence working in research teams.

On the first day of class, Professor P asked the students to read the first chapter of the text and reflect on what questions they had and what was important to them. He used the emails they sent in response to assign initial working groups based on who the students in the room were, a process model strategy. This technique, combined with student journaling and problem contexts driven by student interests about local environmental concerns was Professor P's way of moving into culturally responsive pedagogy. The four-person student working groups Professor P created by putting together students whose responses were the closest were homogeneous by self-described ability and attitude. Students stayed in these working groups for the first half of the term. When the time came for determining working groups for the final project, three of the eight groups did not change while the other five groups rearranged their memberships.

Assessment. Students completed traditional mathematics assignments, such as working on computational exercises, in class. Outside of class student work consisted of the research, analysis, and writing of results completed by the working group on topics chosen by each group. Choices were largely based on the assigned readings in the textbook and on data sets culled from on-line sources (e.g., the Quantitative Environmental Learning Project web data sets). Every week, each student group gave oral presentations on their weekly problems. After five weeks, the students had completed assignments covering the basic tools of differentiation and integration and took a midterm exam. The midterm exam assessed students' basic skills and ability to apply these skills to analyze raw data. A student could take the midterm up to three times, repeating problems similar to the ones that had not been answered effectively on the previous version. "Passing" was defined as the ability to correctly complete at least 90% of the exam. All students passed the midterm, with two students using all three attempts to do so.

Attendance was part of the course grade because group and whole-class discussions about mathematical concepts could only happen if students were present. A student who had three or fewer absences, passed the midterm exam, and completed the project and poster session, earned at least a C in the course. Higher grades were possible through the final project and poster. Lower grades were possible through poor attendance or not passing the midterm exam. Of the 32 students who started the course, 5 dropped during the first week and 27 finished the course; 18 students passed the course with an A, 7 earned a B, and 2 students ended the course with a C. These values for the given term were close to the averages over the life of the course: from 1999 to this writing, the distribution has been 50% A, 25% B, 10% C, and 15% drop the course.

The Final Project and Poster. During the second half of the course, students designed, gathered data, analyzed data, and worked on their final project reports and group posters. First, to choose the final project, each student gave a brief oral description in class of the topic most interesting to her or him. Students then arranged themselves into interest groups. For the final project, each student analyzed a unique set of data and wrote an individual report. The working group structure was there to support the exploration of the topic and to provide a network of others also interested in the same topic to help address any mathematical challenges. Though the interest groups did not write project reports together, each interest group was responsible for creating a poster about their shared topic. During the two-hour final exam period, the class had a poster session. Each group stood next to their poster, explaining their topic and approaches to guests. Members of the faculty, graduate students, and undergraduate students in the mathematics department attended the poster session. In addition to the graded assignments, students also kept journals that were regularly reviewed by the graduate assistant.

Student Journaling

Students wrote in their journals about how the course was going. This allowed the instructor to assess and respond to the students' experiences of the various teaching and curriculum strategies. Below, we summarize the main points that arose in the student journals. The word "many" indicates at least half, "some" is between one-quarter and one-half of students, and "a few" refers to reports by one-quarter or fewer of the students.

Student Comments on Course Processes. More than 65% of students reflected at least once in journaling on how the course processes affected their learning. Early in the term, some suggested a component from the *transmission* or *product* models such as a lecture, scheduled lessons on particular topics, or articulation of specific product-based expectations, might help improve the course:

> I think that the idea of solving problems through a group is a good technique and very refreshing…however, I think that if the class could all come together and learn something current in the news or just to learn something or apply what we've learned to current events – kind of like the article projects – but *with more teacher explanation*, that could summarize all the facts. I don't know that my idea is good, but something is missing from the class. (Week 3, Student H, italics added)

Much of the frustration students reported feeling about the class seemed to be related to their discomfort with the discussion and working group-based learning environment. That is, just as a student in a transmission-based course might be concerned with the constraint of having to accept the rules of mathematics to learn the *content*, students in Environmental Mathematical Modeling found themselves having to "just go with" the non-transmission *processes* of the course:

> This class seems pretty challenging and frustrating right now. I feel like we have no guidance and it gets hard sometimes. But in a way this is just a philosophy of teaching that I am unaware of so I guess I need to just go with it and accept it. (Week 8, Student K)

Student Perceptions of Group Work. The highlight of the course for many students was the group work. For some students who entered the class feeling anxiety about mathematics, working with peers in the class helped to validate and ease their concerns:

> The small group structure really helps the learning level. My other classes average 300 students! I tend to feel really lost when I go to those classes. (Week 4, Student L)
>
> The best part about working in groups for the complete term is the friendships that develop in the process. (Week 10, Student F)

Some students also reported that they learned from their peers when they worked so closely with other students on homework and group projects:

> Today after splitting into groups some of the math concepts finally made some sense to me. We all (at my table) were a little confused with some of

> the math, but once we began talking and helping each other it started to make sense. (Week 2, Student K)
>
> …I must say I like the group interaction. Working in teams is always interesting but group dynamics can sometimes lead to the best of solutions or the most creative ideas. I even see that the groups who were somewhat apprehensive about doing the calculus or just the math itself are very involved in the problem solving effort. This certainly shows progress as far as the class goes. (Week 3, Student G)

The varying levels of mathematics abilities in the class provided a unique challenge for student groups. Although students generally worked in groups with students of similar abilities, often the students with calculus backgrounds provided support for students learning the concepts for the first time:

> In class today we worked in large groups on the class assignments – some students knew how to do it while others seemed overwhelmed. It was nice how the students who knew helped those who didn't. (Week 2, Student H)
>
> Instead of blowing off studying for it [the midterm] and daydreaming in class, I decided to try to help [other students] with their preparation. This was very satisfying to me to help them. (Week 6, Student D)

In the Winter and Spring terms following the Fall term teaching experience described above, the instructor taught traditional calculus. It seemed that there were far more questions from students about the applicability of the material: What use was the exponential function? Why should anybody care about the Taylor series? Questions like this never came up in Environmental Mathematical Modeling, where students had a much better sense of how calculus could be used to make sense of the world. Each calculus topic was introduced as a tool to solve a particular environmental question, at which point the students used calculus to solve a problem they had identified, a problem that was their own.

Extended Example: Culturally Responsive Liberal Arts Mathematics

In this example, we offer a snapshot of Professor H's attempts to move from process- to praxis-based instruction in the context of a first-year liberal arts mathematics course at a medium sized U.S. university (11,000 students). Given the importance of community in culturally responsive curriculum and instruction, we situate Professor H's two praxis-based class sections in the context of the department and of the course – a total of 6 instructors taught 11 class sections. Each liberal arts mathematics class met 150 minutes per week, either 50 minutes per day on Monday, Wednesday, and Friday or 75 minutes per day on Tuesday and Thursday. The two praxis-based sections taught by Professor H met on Tuesday/Thursday.

Course Design

A two-year piloting process for culturally responsive design of the course and choice of curricular materials preceded the implementation described here. At the time of our example, the textbook had been in use for two semesters by Professor H and by three of the other five instructors teaching the course. As was demanded by the administrative rules at the university, the syllabus outlined course content. Unusual to the standard format at the university, the syllabus also outlined several course processes (e.g., writing about mathematics, providing students access to at least three representations for each concept such as words, formula, graph) and instituted a “mandatory choice” of content. Five-sixths of the course content was prescribed in the syllabus through reference to topics covered in the chosen textbook while the remaining one-sixth of the course content, approximately 2 weeks of class, was to be negotiated on a semester-by-semester basis in each class. Each instructor chose – with or without student input, depending on the instructor’s teaching style – some additional topic or topics on which to focus for the two week “choice” part of the course.

Of the six instructors teaching in the Spring term considered here, one was culturally responsive praxis-oriented (Professor H, a PhD in mathematics), two expressed views and curricular planning that aligned with a process-based view (both with Master’s degrees in mathematics, one with a culturally responsive approach), and three were firmly transmission-based (one a PhD in mathematics, one with a Master’s degree in mathematics teaching, and one with a Master’s degree in mathematics; the last with a culturally responsive approach). During the semester, all instructors met with each other in a coordination seminar every week for an hour to make decisions about course material, to exchange ideas for quizzes, exams, ways of “presenting the material,” and to write and fine-tune four problems that would be common to all the liberal arts mathematics final exams. Though the four final exam items were in common, the rest of each final exam was determined however the teacher saw fit, as were all other assessments in the course.

Selecting Course Content

As noted, the privilege and obligation of choosing some of the course content was left up to students and/or the instructor. In the given term, one of the process-based and two of the transmission-based instructors made the content choice. In the other sections of the course, the choice was made by students alone by a vote or, in Professor H’s classes, as the result of an in-class discussion among students and a negotiation between student-elected representatives and the instructor. At the level of curricular materials, then, many students had at least one extended, two-week, opportunity for appropriation and feedback.

Course Processes

While in the first extended example we gave an idea of responsiveness through a broad examination of course design, in this second example we give a brief overview of course process and strive to illustrate some of the depth of responsiveness through attention to activities around a particular concept. At the level of community knowledge in Professor H's praxis-based sections, students posed and solved problems in groups during class. They also completed worksheets in class made up of problems written by students in other sections (in the same and previous terms). In- and out-of-class assessments included quizzes, exams, and presentations about brief and lengthy projects.

Problem-Posing. By the third problem-posing session, in Week 5, students had begun to discuss the problems they posed with the future solver, such as a fellow student, in mind. Four-member student item-writing groups exchanged items and explicitly referred to the culturally rich context of the future problem solver in offering critique and clarification for problem statements. These problem posing activities evolved over the semester from "Pose a similar problem for a fellow student in another liberal arts mathematics class" to "Write a similar problem in a real-world context that would be appropriate for an older student returning to college after a 5 year absence due to service in the armed forces" to "Write an item that gets at the idea of rate of inflation that would be appropriate for a middle class, European American pre-service teacher who is learning about ratio and proportion. Write a second item that she could use next year that would be accessible to her socio-economically and ethnically diverse pre-algebra students." This final problem-posing prompt occurred in the last week of the course.

Activities. Professor H's praxis-based course also included short activities to increase student awareness of the classical and societal learning and teaching cultures around them. For example, early in the semester, students read, discussed, and applied Smith and Stein's (1998) Mathematical Task Analysis Guide (TAG) and Mathematical Tasks Framework. One class-time activity included a quick review of textbook problems and online homework items by students using the TAG and led to their noting that the textbook problems were largely Procedures With Connections and Doing Mathematics while online items were almost exclusively Memorization and Procedures Without Connections tasks. Similarly, an introduction of the concepts covered by the words "scaffolding" and "funneling" gave students vocabulary they used throughout the term in communicating with each other and with the instructor about the nature of their own and others' learning.

Focus on Content: Consumer Price Index

The topic of Consumer Price Index (CPI) and the calculation of inflation rate as relative change in CPI[3] were covered by all 11 liberal arts mathematics sections for at least 50 minutes of class meeting time. In Professor H's praxis-based classes, the 50 minutes was made up of 20 minutes on one day and 30 on the next day of class meeting. Professor H started by asking students what inflation was and how you might measure it in the changing price of a candy bar. Within 5 minutes students generated a relative change formula and the instructor used it to introduce how CPI is derived, work two examples of translating one year's value to another, and one of finding rate of inflation (7 minutes); after this students worked in pairs on problems from the book on changing dollar-years and finding rates of inflation (7 minutes). In the next class meeting, students posed new problems (20 minutes) and critiqued posed problems (10 minutes). In all other sections, the instructors devoted one class meeting to the topic. Students in these classes took notes during a lecture and, in one process-based section, worked in groups on two problems in class while the praxis-based students completed seven problems from the text and posed two new items during the 50 minutes of time spent on the topic in class.

In Professor H's praxis-based course, the topic was spread across two class meetings to allow students to investigate – outside of class – the notion of CPI in two ways. First, by completing six on-line homework skill-building problems and secondly, by creating their own CPI after compiling price information by either: (a) visiting two supermarkets: one in a largely working-class Latino neighborhood and one in a predominantly white middle-class neighborhood or (b) comparing supermarket prices from a local English language newspaper and a local Spanish language paper (most students chose (a)). Though none of the other classes did such a project, students in the nine transmission- and process-based class sections had 4 to 12 homework problems from the textbook, and 4 to 8 on-line problems that were all to be completed after the one class meeting in which CPI was addressed.

A paragraph at the end of the textbook chapter on CPI introduced the idea of a Health Care CPI; however, none of the instructors brought it up in class and none assigned homework on it. Consequently, the exam item discussed below was probably a novel problem for most students. The praxis-based students' cultural responsiveness to and awareness of the world through mathematics can be seen, in part, in the answers they offered on the test question. One of the four common final exam items in the course was CPI-based[4]:

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[3] For example, the rate of inflation from 1990 to 2000: $(CPI_{2000}-CPI_{1990})/CPI_{1990}$.

[4] The common final exam items were written and unanimously approved by all six instructors.

The overall Consumer Price Index (CPI) in September 2002 was 185.1 and the CPI for September 2004 was 189.7. Meanwhile, the Health Care CPI for September 2002 was 308.6 and for September 2004 it was 324.0 (Source: www.bls.gov/news.release/cpi.nr0.htm).

(a) What was the overall rate of inflation from September 2002 to September 2004?

(b) What was the rate of inflation for Health Care from September 2002 to September 2004?

(c) Compare the rate of inflation for Health Care to the overall rate of inflation. Please write using complete sentences.

Among the 270 students in the *transmission*- and *process*-based sections of the course, approximately 65% gave correct numerical answers to parts (a) and (b) and all but one of these answers was notably similar to the solution shown below:

a. 2.5%

b. 5%

c. Health care is double the rate of inflation.

In the two *praxis*-based sections of the course, 89% of the 64 students provided correct numerical answers to both (a) and (b), written using complete sentences, of the form:

a. The rate of inflation between September 2002 and September 2004 was about 2.5%.

b. The rate of inflation for Health Care between 2002 and 2004 was about 5%.

Many student answers to (c) in the two praxis-based sections of the course were quite different from the answers offered by students in the other sections. Though about 40% gave answers to (c) like the one shown above, 50% of the praxis-based students provided answers to part (c) that took a form similar to one or more of the following:

c_1. *Since the health care inflation rate was double the overall rate (and assuming healthcare is part of the CPI), there must be other things that had much lower rates of inflation to balance the large increase in health care costs.*

c_2. *The fact that the cost of health care rose at twice the rate of inflation means that medical care will take a larger portion of a family's income than other goods and services.*

c_3. *Health care during those years cost more than twice what it should have, so people with lower incomes who might get a 2.5% cost of living raise would not get enough of a raise to afford the same quality of health care they could in 2001.*

Students offering answers like c_1 were explicit about their assumptions regarding *how* CPI was determined. They interrogated the idea of CPI, rather than simply accepting it, and then drew mathematically logical conclusions about Health Care CPI based on those articulated assumptions.

In addition to stating the straightforward comparative information, that one was double the other, students providing answers like c_2 called upon the value of the mathematical constructs of CPI and inflation to justify assertions about the larger world. That is, they used mathematical computation and logic to support a conclusion about economic realities.

Similarly, students who gave answers like c_3 noted the comparative ratio and extended the idea, mathematically. Many talked about the relative impact of the different inflation rates on different sub-populations. In the example shown, the student focused on the life-quality inequities arising from an economic inequity in cost-of-living allotment.

The number of sections in the text read, number of homework problems posed, reviewed, and correctly completed, and depth and scope of projects completed by students was highest in the praxis-based sections and lowest in the process-based sections with the transmission-based sections in between. The scores on the common final exam items (scored independently of instructor grading, using a uniform rubric) were slightly higher in the praxis-based sections and about the same in transmission- and process-based sections.

Discussion

As in all culturally responsive *process* and *praxis* model curricula, student sense-making and communication were important components in both of the extended example courses. The proxy measurement for student sense-making and communication in the process-based environmental calculus course could be found in the significance of attendance in determining course grade and the reliance on working in groups. The ability to work as a member of a team is a valuable skill for many college graduates and is responsive to the needs of one entering the middle-class working world. In environmental calculus, the course final project built student skills in examining the status quo and situating work within it, both indicators of a process view. Also, the poster brought in a communication technique valued in the mathematical culture. The culturally responsive aspects here included the focus on academic achievement as witnessed by Professor P, peers, and other faculty (in the poster session), environmental consciousness and critique, community building and personal connections, and attention to individual self-worth and abilities. Still under development for Professor P was how to operationalize the culturally responsive tenets of community knowledge, social consciousness and critique, cultural affirmation, competence, and exchange, in mathematically rich ways.

Indicators of student sense-making in the praxis-based liberal arts classes included the twice monthly problem-posing sessions where students worked in small groups. In the liberal arts mathematics course, students regularly examined and mathematized cultural, socio-economic, and political contexts and looked for alternate solutions to status quo answers – indicative of the transformative presumptions of a culturally responsive praxis approach. Still under development for Professor H was how to support students in sustaining their development as culturally responsive learners outside of and beyond their enrollment in the liberal arts mathematics class.

In the environmental calculus class, students' journal writing made clear the ways in which they were appropriating the course content and processes and was a channel for feedback to the instructor and graduate assistant of the course. Professor P's intention was to balance attention to socially important environmental issues with fostering *legitimate peripheral participation* in the mathematical community of practice of academe (Lave & Wenger, 1991). However, this intention was only partially realized. Having students choose and work with real-life data sets to answer questions and address issues in environmental science was seen as valuable and legitimate by the students. In this sense, the course was responsive. However, most members of the academic mathematics community would not be likely to say that what the students were learning was mathematics. Rather, a "pure mathematician" would identify the student's work as applied mathematics. The course content and processes did not support abstract reasoning about derivative and integral as mathematical constructs. In this sense, student participation was indeed "peripheral" to the mathematics community, but "illegitimate" to that community (Barton & Tusting, 2005), which troubled Professor P. Attempting to negotiate the tension between the demands of classical and societal knowledge may often lead to such situations. Relying on community and critical knowledge as a foundation, Professor H situated mathematics learning in service to multiple global demands of which the academy and society were two parts, also a potentially "illegitimate" form of mathematical participation in the habitus of the field of academic mathematics.

Culturally responsive curriculum and instruction offer opportunities for learning through a wide array of culturally authentic mathematical and pedagogical contexts, whether or not the contexts are from one's personal community knowledge. For example, teaching a largely European American, middle-class, group of students through contexts that are personally relevant *and* through those that are socio-culturally rich but not echoes of personal experience allows a diminishing of the perceived "other-ness" of those not from white, middle-class backgrounds (Rodriguez & Kitchen, 2005). In particular, such duality of instruction can allow students the opportunity to see their own familiar culture as "other," that is, as a culture rather than as a neutral background (Spindler, 2000).

College mathematics instructors operate at the nested intersection of several fields. College mathematics teaching lives within the larger field of academic mathematics, which itself is nested within the larger field of post-secondary education. By analyzing the habitus of agents (students, teaching colleagues, administrators), a college mathematics instructor can inform teaching decisions with pertinent community and classical knowledge. Making such analyses and using the information to change one's instruction can be dangerous to a career. Taking on culturally responsive pedagogy and attending to community and critical knowledge growth for students is a political act. However, engaging in that political act need not be a radical or abrupt move. It may start with noticing the existence of students' funds of knowledge and working to uncover and include community knowledge in framing mathematical activities. Next, one might make efforts to regularly have students become the framers of activities. A subsequent step can be inviting students to seek, with the instructor, authentic and relevant problems whose solutions may be supported by expertise in abstract and applied mathematics. Helping students to build expertise with abstract concepts while also learning how to apply them may be facilitated by being explicit with students that the instructional goal is the co-development of both kinds of understanding.

Learners walk into a classroom with myriad forms of habitus, including a variety of intellectual, personal, social, political, and economic resources. Most college students have learned, as they were learning mathematics, that they are expected to suppress any contribution they might be tempted to make to context. The importance of isolation and individuality having been established, students are likely to have internalized the notion that to learn mathematics they must sacrifice agency and intuition to the rules of mathematics – both as a discipline and as a classroom milieu (Boaler & Greeno, 2000). Recognizing and confronting with students this part of their habitus is one of the challenges of culturally responsive pedagogy. It is the creation of a caring environment that can facilitate change for students and for the instructor.

Responsiveness means understanding and interacting with people in context, including the variation in how that context may be created and perceived by all parties involved. Instructors' or curriculum designers' *intentions* for students' experiences of mathematics is not "context" in the sense used here. Context is mutually defined and emergent rather than wholly pre-determined. It relies on the dynamics of human experience, perception, and on the interaction of the people in the room. Most of the curricular materials (from instructor-created exams to mass market textbooks) for collegiate mathematics disregard interpersonal or communal cognitive activity. With the possible exception of some reform calculus and quantitative reasoning books, college textbook development does not include addressing contextual potential – the variability in how instructors and students might want to use the books.

Conclusion

Cultural responsiveness both informs and empowers. Like the process model, a culturally responsive approach values the contributions students, colleagues, societies and policies can make to course content. At the same time, like the praxis model, culturally responsive pedagogy values the contributions that knowledge of content can make to improving society, policy, and individual lives.

Culturally responsive college mathematics curriculum and instruction foster a classroom environment where it is safe, though perhaps not especially comfortable, to engage deeply with mathematical and pedagogical ideas. In the *transmission* view, instructors can be responsive to students by providing information through multiple modes and stating the connections among them (e.g., multiple representations: written, spoken, graphical, tabular; or multiple presentations: snapshots and animations). In the *product* view, the types of work students seek to prepare themselves for can shape the content of examples, including student-generated examples, and the forms of assessment. In the *process* view, each student can be overtly encouraged to establish personal relevance for course material while constructing a profound understanding of certain mathematical ideas. In the *praxis* view, students and instructors can explore, disagree, and come to consensus while both mastering standard representations for concepts and implementing that understanding in ways relevant to the academy, society, community, and globe.

We close with our list of six ways culturally responsive pedagogy is enacted in college mathematics, through:

- Validating the experiential capital and approaches to learning that students bring to the enterprise of learning in a college mathematics course. Validation happens when teacher and students find out about and accept each other's perceptions and then fold this awareness into resources and materials that are rich with classroom, multicultural, multiple ability, and multiple social-class connections. Explicit efforts to build bridges between socially and mathematically privileged university professors and their students can serve as a model for students' own culturally responsive interactions with others. That is, the classroom community overlaps the research mathematics community and informs a variety of cultural repertoires for students and instructor.

- Empowering students as citizens, as learners, and as teachers (for themselves and others) through implicit and explicit support and development of self-regulation and socially aware critical thinking; this overlaps the idea of teaching for social justice.
- Supporting the development of awareness among students of the knowledge, skills, and value sets – including understandings of mathematical concepts – associated with access to social, economic, and political power. In particular, codes of power, the mainstream mathematical rules and communication practices (Delpit, 1996), are explicitly discussed by students and teacher until a repertoire is established about how to translate between cultural worlds.
- Comprehensively teaching the whole learner through explicitly recognizing and valuing of the diverse ways that cultural and personal identities mediate ways of cognitive engagement as well as explicitly addressing multiple modes of learning in instructional design. In collegiate mathematics service courses this can be as simple as presentations and activities that use the multiple representations of formulas, words, tables, and graphs (common in reform calculus textbooks).
- Engaging in multidimensional assessments of learning. Such evaluations of learning in mathematics can include reflective and explanatory writing, portfolios, group-grade collaborative assignments, individual-grade cooperative assignments, projects, discussions, and peer- and self-evaluated work.

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